

Strained Layer Crystalline Undulator

A. Kostyuk

Ultrarelativistic charged particles are predicted to emit hard electromagnetic radiation of undulator type while channeling in a crystal with periodically bent crystallographic planes. The recently proposed crystalline undulator with a bending amplitude smaller than the distance between the bent planes and a bending period shorter than the period of channeling oscillations is far superior to what was proposed previously. In the same time, it is more challenging from the technical point of view because its bending period has to be in the sub-micron range. It is shown that a mixed crystal of silicon-germanium with properly varying germanium fraction can have the necessary bending parameters. Moreover, it is predicted to be stable against misfit dislocations.

PACS numbers: 68.65.Cd, 46.25.Cc 61.72.uf, 61.85.+p

Channeling [1, 2] of ultrarelativistic charged particles through a periodically bent crystal can be used to generate hard electromagnetic radiation of undulator type [3, 4]. Such a radiation source may become a compact and affordable alternative to presently existing synchrotron radiation sources that are widely used in science and technology [5] but are very big and expensive. Moreover, a coherent radiation source, a hard X ray or gamma ray laser, can be built on the basis of the crystalline undulator [6].

Initially, it was suggested that the charged projectiles should follow the sinusoidal shape of the bent crystallographic planes of the crystalline undulator performing nearly harmonic transverse oscillations. Due to these oscillations, the spectrum of the electromagnetic radiation emitted by the particles in the forward direction was expected to have a narrow peak, similarly as it took place in an ordinary (magnetic) undulator [7–9].

Several conditions had to be satisfied by the crystal and the beam to reach the desirable properties of the spectrum [10]. In particular, the bending period had to be much larger than the period of channeling oscillations, $\lambda_u \gg \lambda_c$. Otherwise the projectile would not follow the shape of the bent crystal channel. Additionally, the bending amplitude should be much larger than the distance between the guiding crystallographic planes, $a_u \gg d$, to make sure that the spectrum is dominated by the undulator peak rather than by the channeling radiation, which is present also in the case of a straight crystal [11]. In the following, such a crystalline undulator will be referred to as LALP CU (**L**arge **A**mplitude and **L**ong **P**eriod **C**rystalline **U**ndulator).

A new type of crystalline undulator has been proposed recently [12]. It has been demonstrated by numerical simulations that a crystalline undulator with a bending period smaller than the channeling period,

$$\lambda_u < \lambda_c \quad (1)$$

and the bending amplitude smaller than the channel width

$$a_u < d \quad (2)$$

has several advantages over LALP CU. In particular, it has a much larger effective number of undulator periods and it requires much lower beam energy for the production of radiation of the same frequency. The new type of crystalline undulator will be referred to as SASP CU (**S**mall **A**mplitude and **S**hort **P**eriod **C**rystalline **U**ndulator). The channeling projectile in the SASP CU does not follow the shape of the bent planes. Still, the shape of the trajectory does contain a Fourier component with the period equal to that of the undulator bending. As the result, a narrow undulator peak is present in the radiation spectrum. The frequency of the undulator radiation is higher than that of the channeling radiation. Therefore, the intensity of the undulator peak is higher than the channeling one despite of the smallness of the undulator amplitude with respect to the channel width (see [12] for details).

From technological point of view, SASP CU is more challenging than LALP CU. For moderate projectile energy $E \lesssim 1$ GeV, it is necessary to bend the crystal with a period shorter than one micron. The purpose of the present letter is to analyze whether the manufacturing of such crystals is possible within presently existing technologies.

Several techniques have been proposed to produce LALP CU. At the very beginning [3, 4], it was suggested to use ultrasonic waves to bend the crystal. However this idea appeared to be too challenging from the technical point of view and, therefore, it is still waiting for its experimental implementation. The attenuation of the acoustic wave is inversely proportional to its wavelength. i.e. the shorter the undulator period, the stronger is the attenuation. Therefore, using ultrasound in the case of the SASP CU would be even more difficult or even impossible.

A few other technologies utilize the idea of imposing periodic stresses on the surface of the crystal sample. It can be accomplished by making regularly spaced grooves on the crystal surface either by a diamond blade [13, 14] or by means of laser-ablation [15]. Alternatively, the stress can be imposed by deposition of a different material on the surface of the crystal. It was initially proposed

to use crystalline materials with similar but slightly different lattice constants [16]. Later it was found more practical to deposit Si_3N_4 layers onto the surface of a silicon crystal [14]. Deposition takes place at high temperature. The stress appears after cooling the crystal to the room temperature due to different coefficients of thermal expansion of the crystal and the strips.

Manufacturing of SASP CU with the surface stress technology is hardly possible for the following reasons. First, the strain that is produced by the surface stress decreases fast with the distance from the surface. Therefore, the crystal dimension in one of the two transverse directions has to be of the order of the undulator period λ_u [17], which is in the sub-micron range for moderate beam energies $E \lesssim 1$ GeV due to (1). Preparing such a thin crystal and depositing sub-micron sized strips on its surface, not to mention making regularly spaced grooves, are highly problematic. Second, the bending amplitude varies strongly across the crystal. Only the most central part of it having the width of $\sim \lambda_u/(2\pi)$ has nearly constant bending amplitude [17]. Only this part should be exposed to the beam. This means that the size of the beam spot in the corresponding transverse direction has to be in the range of tens of nanometers and the crystal has to be placed with the corresponding accuracy. Moreover, as in any other channeling experiment, the beam divergence at the entrance to the crystal should not exceed the critical (Lindhard's) angle, which is typically in the range of a few hundreds microradian or smaller. Therefore, the transverse emittance of the beam should not exceed several nm-mrad. None of the existing electron or positron accelerators has such high beam quality [18].

Fortunately, there is one more method of crystal bending which is free from the above flaws. Growing crystals with varying chemical composition [19] creates strain inside the crystal volume rather than on its surface. Therefore, there is no such severe restriction on the crystal size as in the previous case. The bending amplitude does not vary across the crystal. Hence the size of the beam spot can be in the range of hundreds of microns or even a few millimeters in both transverse directions. This allows for using moderate quality beams at presently available accelerators.

The most mature of such technologies is using the method of molecular beam epitaxy for growing $\text{Si}_{1-\chi}\text{Ge}_\chi$ crystals with periodically varying germanium fraction χ . Such heterostructures were intensively studied for the purposes of the semiconductor industry. There is rather extensive practical experience of creating $\text{Si}_{1-\chi}\text{Ge}_\chi$ strained layer of various thickness from 10 Å to microns or even tens or hundreds of microns. Their properties are rather well known (see, for instance, [20–22] and references therein). Deflection of the beam by $\text{Si}_{1-\chi}\text{Ge}_\chi$ strained layer crystals has been studied experimentally, see e.g. [23, 24]. A $\text{Si}_{1-\chi}\text{Ge}_\chi$ LALP CU is being used in

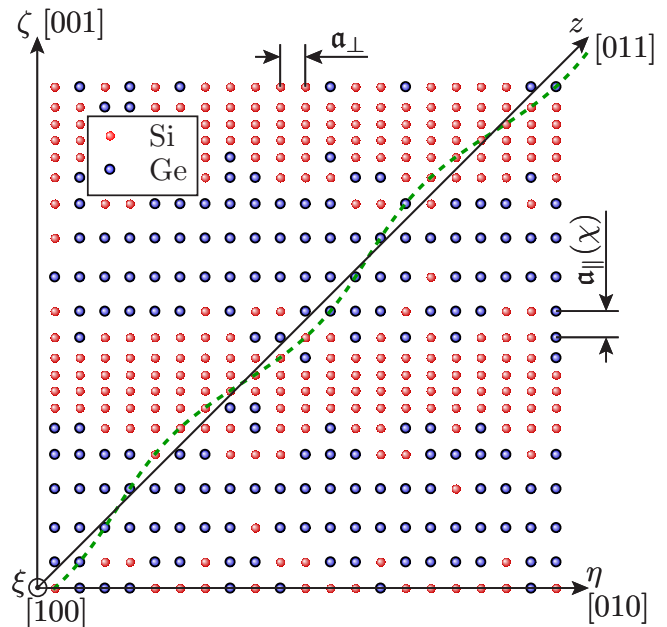


FIG. 1: A strained layers superlattice of gradually varying chemical composition. A higher fraction χ of germanium results in a larger longitudinal lattice constant $a_{||}(\chi)$. Due to the periodic variation of χ , the crystal axis $[011]$ (the dashed line) is periodically bent. So is the plane $(01\bar{1})$, which is perpendicular to the plane of the figure and contains the axis $[011]$. For illustration purposes, the variation of the lattice constant is strongly exaggerated, while the bending period is shown much shorter than it should be in the reality. To simplify the figure, only one atom per cubic cell is shown.

ongoing experiments at Mainz Microtron [25]. Therefore, the further discussion will be focused on this crystalline material. The idea of [19] will be studied in greater details in order to show that it can be used to produce a SASP CU.

Let us consider a $\text{Si}_{1-\chi}\text{Ge}_\chi$ single crystal that was grown in the direction $[001]$ with varying germanium content χ , see figure 1. The coordinate axes ξ , η and ζ are chosen to coincide with the crystallographic axes $[100]$, $[010]$ and $[001]$, respectively. This means that the germanium fraction χ depends on ζ but does not depend on ξ and η .

Let us take an element of the crystal volume containing \mathcal{N} layers of elementary cells in each direction. The volume has to be large enough $\mathcal{N} \gg 1$ so that the elasticity theory can be applied. On the other hand, it should not be very large so that the variation of Germanium concentration χ could be neglected within the volume.

Being relaxed, the volume would have a cubic shape with the edge length $\mathcal{N}a(\chi)$. The 'native' lattice constant at given χ , $a(\chi)$, can be found from a linear interpolation

(Vegard's law [26])¹ between the lattice constants of pure silicon \mathbf{a}_{Si} and germanium \mathbf{a}_{Ge} :

$$\mathbf{a}(\chi) = (1 - \chi)\mathbf{a}_{\text{Si}} + \chi\mathbf{a}_{\text{Ge}}. \quad (3)$$

If no defects are present in the crystal, the transverse positions (ξ, η) of the atoms in each crystal layer have to coincide with those of other layers containing a different fraction of germanium. For this reason, the actual transverse dimension of the volume element $\mathcal{N}\mathbf{a}_{\perp}$ is, generally speaking, different from $\mathcal{N}\mathbf{a}(\chi)$ and does not depend on χ . Therefore, the volume element is deformed.

The transverse deformation induces a longitudinal one. Hence, the longitudinal size of the volume element, $\mathcal{N}\mathbf{a}_{\parallel}(\chi)$, is also, generally speaking, different from $\mathcal{N}\mathbf{a}(\chi)$. In contrast to \mathbf{a}_{\perp} , the longitudinal size of the volume element does depend on χ .

The diagonal elements of the strain tensor describing the deformation are

$$\epsilon_{\xi\xi} = \epsilon_{\eta\eta} = \frac{\mathbf{a}_{\perp} - \mathbf{a}(\chi)}{\mathbf{a}(\chi)}, \quad (4)$$

$$\epsilon_{\zeta\zeta} = \frac{\mathbf{a}_{\parallel}(\chi) - \mathbf{a}(\chi)}{\mathbf{a}(\chi)} \quad (5)$$

No shear deformation is present.² Therefore non-diagonal elements of the strain tensor are zero.

Only three elements of the stiffness tensor c are independent in the case of crystals with the cubic symmetry (see e.g. [28]). Only two of them, $C_{11} = c_{\xi\xi\xi\xi} = c_{\eta\eta\eta\eta} = c_{\zeta\zeta\zeta\zeta}$ and $C_{12} = c_{\xi\xi\eta\eta} = c_{\xi\xi\zeta\zeta} = c_{\eta\eta\xi\xi} = c_{\eta\eta\zeta\zeta} = c_{\zeta\zeta\xi\xi} = c_{\zeta\zeta\eta\eta}$ are relevant to the present analysis. Hence, Hook's law for the deformation (4) and (5) has the following form

$$\begin{aligned} \sigma_{\xi\xi} = \sigma_{\eta\eta} &= (C_{11}(\chi) + C_{12}(\chi)) \frac{\mathbf{a}_{\perp} - \mathbf{a}(\chi)}{\mathbf{a}(\chi)} \\ &+ C_{12}(\chi) \frac{\mathbf{a}_{\parallel}(\chi) - \mathbf{a}(\chi)}{\mathbf{a}(\chi)}, \end{aligned} \quad (6)$$

$$\begin{aligned} \sigma_{\zeta\zeta} &= 2C_{12}(\chi) \frac{\mathbf{a}_{\perp} - \mathbf{a}(\chi)}{\mathbf{a}(\chi)} \\ &+ C_{11}(\chi) \frac{\mathbf{a}_{\parallel}(\chi) - \mathbf{a}(\chi)}{\mathbf{a}(\chi)}. \end{aligned} \quad (7)$$

The stress tensor σ has to satisfy the following conditions to ensure the mechanical equilibrium of the crystal

$$\overline{\sigma_{\xi\xi}} = \overline{\sigma_{\eta\eta}} = 0 \quad (8)$$

$$\sigma_{\zeta\zeta} = 0. \quad (9)$$

The overline stands for averaging over ζ , e.g.

$$\overline{\sigma_{\xi\xi}} = \frac{1}{L_{[001]}} \int_0^{L_{[001]}} \sigma_{\xi\xi} d\zeta. \quad (10)$$

Here $L_{[001]}$ is the crystal dimension along the crystallographic axis [001].

The elements of stiffness matrix, $C_{11}(\chi)$ and $C_{12}(\chi)$, depend on the germanium concentration χ . Because the mechanical properties of silicon and germanium are rather similar, this dependence can be neglected. The average values $\overline{C_{11}}$ and $\overline{C_{12}}$ will be used in the following.

From (7) and (9) one obtains

$$\mathbf{a}_{\parallel}(\chi) = \mathbf{a}(\chi) - 2 \frac{\overline{C_{12}}}{\overline{C_{11}}} (\mathbf{a}_{\perp} - \mathbf{a}(\chi)) \quad (11)$$

Substituting the last expression into (7) and (9) yields

$$\left(\frac{\mathbf{a}_{\perp} - \mathbf{a}(\chi)}{\mathbf{a}(\chi)} \right) = 0. \quad (12)$$

The mismatch between the lattice constants of silicon and germanium is small as well. Therefore, the variation of $\mathbf{a}(\chi)$ in the denominator can be also neglected. Hence, one obtains from (12) and (3)

$$\mathbf{a}_{\perp} = \overline{\mathbf{a}} = (1 - \overline{\chi})\mathbf{a}_{\text{Si}} + \overline{\chi}\mathbf{a}_{\text{Ge}}. \quad (13)$$

Substituting the last expression into (11) one obtains

$$\mathbf{a}_{\parallel}(\chi) = \overline{\mathbf{a}} + \tilde{\chi} \left(1 + 2 \frac{\overline{C_{12}}}{\overline{C_{11}}} \right) \Delta \quad (14)$$

with

$$\tilde{\chi} = \chi - \overline{\chi} \quad (15)$$

and

$$\Delta = \mathbf{a}_{\text{Ge}} - \mathbf{a}_{\text{Si}}. \quad (16)$$

The angle δ between the axes [010] and [011] satisfies the following equality

$$\tan \delta = \frac{\mathbf{a}_{\parallel}(\chi)}{\mathbf{a}_{\perp}} = 1 + \tilde{\chi} \left(1 + 2 \frac{\overline{C_{12}}}{\overline{C_{11}}} \right) \frac{\Delta}{\overline{\mathbf{a}}}. \quad (17)$$

Let us define the coordinate axis z as the average direction of the bent crystallographic axis [011]. Axis z makes the angle $\pi/4$ with the axis η . Therefore, the angle between the axis z and the bent crystallographic axis [011] is

$$\tilde{\delta} = \delta - \pi/4. \quad (18)$$

Substituting the last expression into (17) and neglecting the terms of the order of $\tilde{\delta}^2$ or higher yield

$$\tilde{\delta} = \tilde{\chi} \left(\frac{1}{2} + \frac{\overline{C_{12}}}{\overline{C_{11}}} \right) \frac{\Delta}{\overline{\mathbf{a}}}. \quad (19)$$

¹ A small deviation [27] from Vegard's law is neglected in the present analysis.

² In fact, shear deformation is present at the edges of (001) planes of the crystal, but it decreases fast with the distance from the edge. It is assumed that the sample is sufficiently large in the directions ξ and η so that the shear deformation can be safely neglected in the bulk of the crystal.

TABLE I: Parameters of silicon and germanium: the lattice constant \mathbf{a} and stiffness coefficients C_{11} and C_{12} [27, 30, 31].

	\mathbf{a} (Å)	C_{11} (GPa)	C_{12} (GPa)
Si	5.431	165.6	63.9
Ge	5.658	126.0	44.0

Let the function $y(z)$ describe the shape of the crystallographic axis [011]. Then

$$\frac{dy}{dz} = -\tan \tilde{\delta} \approx -\tilde{\delta}. \quad (20)$$

Therefore,

$$y(z) = y(0) - \left(\frac{1}{2} + \frac{\overline{C_{12}}}{\overline{C_{11}}} \right) \frac{\Delta}{\bar{\mathbf{a}}} \int_0^z \tilde{\chi}(z) dz. \quad (21)$$

In particular, for χ harmonically varying between 0 and $2\bar{\chi}$,

$$\chi(z) = \bar{\chi} + \tilde{\chi}(z) = \bar{\chi} + \bar{\chi} \sin \left(2\pi \frac{z}{\lambda_u} \right), \quad (22)$$

the shape of the bent axis is also harmonic:

$$y(z) = y(0) + a_u \cos \left(2\pi \frac{z}{\lambda_u} \right), \quad (23)$$

with the bending amplitude

$$a_u = \left(\frac{1}{2} + \frac{\overline{C_{12}}}{\overline{C_{11}}} \right) \frac{\Delta}{\bar{\mathbf{a}}} \frac{\lambda_u}{2\pi} \bar{\chi}. \quad (24)$$

Let us rewrite the last expression in the following form³

$$\bar{\chi} = 2\pi \frac{\bar{\mathbf{a}}}{\Delta} \left(\frac{1}{2} + \frac{\overline{C_{12}}}{\overline{C_{11}}} \right)^{-1} \frac{a_u}{\lambda_u}. \quad (25)$$

The average quantities in the right hand side depend on $\bar{\chi}$. Therefore, (25) is, strictly speaking, an equation which has to be solved for $\bar{\chi}$. Still, $\bar{\chi} = 0$ can substituted to the right hand side in the zero approximation, i.e. the parameters of pure silicon can be used instead of the average quantities.

Using the numerical values from Table I one obtains

$$\bar{\chi} \approx 170 \frac{a_u}{\lambda_u}. \quad (26)$$

³ The term $\overline{C_{12}}/\overline{C_{11}}$ in the parentheses appeared due to longitudinal deformation that is induced by the transverse one (see the two paragraphs after equation (3)). The induced deformation was not taken into account in [29]. It was assumed there that $\mathbf{a}_{\parallel}(\chi) = \mathbf{a}(\chi)$. For this reason, the necessary germanium content was overestimated by the factor of about 1.8.

To find the next-to-leading order approximation, let us assume that elastic constants of the mixed $\text{Si}_{1-\chi}\text{Ge}_{\chi}$ crystal can be approximated by a linear interpolation between the corresponding values for pure silicon and germanium, similar as it was done for the lattice constant (3). Substituting (26) into the right hand side of (25) and neglecting higher order terms, one obtains

$$\bar{\chi} \approx \bar{\chi}^{(0)} (1 + 0.073 \bar{\chi}^{(0)}), \quad (27)$$

where $\bar{\chi}^{(0)}$ is found from (26). One sees that the second order correction is insignificant, especially if $\bar{\chi}^{(0)}$ is small. But even at $\bar{\chi}^{(0)} \approx 0.5$ (which is the maximum possible value of $\bar{\chi}$) the second order correction does not exceed 3.7 %. Therefore, formula (26) is sufficiently accurate for practical purposes.

The bending period λ_u and the amplitude a_u refer to the axial channel [011]. They also valid for the bent planar channel (01 $\bar{1}$), provided that the beam is directed into this channel at a small angle to the axis [011].

It should be stressed that the period of the variation of χ along the direction of crystal growth [001] is smaller than λ_u :

$$\lambda_{[001]} = \frac{\lambda_u}{\sqrt{2}}. \quad (28)$$

Therefore,

$$\chi(\zeta) = \bar{\chi} \left[1 - \sin \left(2\pi \sqrt{2} \frac{\zeta}{\lambda_u} \right) \right]. \quad (29)$$

Formulas (26) and (29) contain all the information which is necessary to manufacture a strained layer crystalline undulator with desired bending parameters λ_u and a_u .

Still, the amplitude a_u cannot be arbitrary large. There is a maximum possible value of a_u at given λ_u . The theoretical limit is set by $\bar{\chi} = 0.5$ which is the maximum possible value of $\bar{\chi}$ corresponding to χ varying between 0 and 1. Substituting $\bar{\chi} = 0.5$ into (24), one obtains

$$a_{\text{th}} = 2.83 \cdot 10^{-3} \lambda_u. \quad (30)$$

However, the theoretical limit usually cannot be reached in practice. It is well known that there exist a critical thickness h_c of a strained crystal layer in crystalline heterostructures. If the layer thickness exceeds h_c , dislocations appear in the crystal lattice that relax the strain. The larger the concentration of germanium in the layer, the smaller is its critical thickness.

To put it differently, there exists a critical concentration of germanium χ_c for a given layer thickness h such that the layer of thickness h is relaxed with dislocations if its germanium fraction exceeds χ_c . Because the bending amplitude is proportional to the germanium concentration, this means that there exists a maximum amplitude of the undulator bending for a given undulator period.

The critical layer thickness for stable crystalline heterostructures were studied in [32–34]. The obtained results differ only slightly. The formula based on the approach of J. W. Matthews and A. E. Blakeslee [33] will be used in the present analysis. The critical germanium concentration for a stable $\text{Si}_{1-\chi}\text{Ge}_\chi$ layer of thickness h grown on a pure silicon substrate is [35]

$$\chi_s = \frac{5.5 \text{ \AA}}{h} \ln \frac{h}{1 \text{ \AA}} \quad (31)$$

(the subscript ‘s’ stands for ‘stable’). To apply this formula to the crystalline undulator, the following two points have to be taken into accounts. First, the transverse lattice constant of the crystalline undulator is not equal to that of the pure silicon. It corresponds to the average germanium concentration $\bar{\chi}$. Therefore the deviation $\tilde{\chi}$ from the average concentration has to be used instead of χ . Second, formula (31) assumes a constant germanium concentration in the epitaxial layer, while $\tilde{\chi}$ varies between 0 and $\bar{\chi}$ (or between $-\bar{\chi}$ and 0) within a half-period. Therefore, the average value over the half period

$$\langle \tilde{\chi} \rangle = \frac{2}{\lambda_{[001]}} \int_0^{\lambda_{[001]}/2} \bar{\chi} \sin \left(2\pi \frac{\zeta}{\lambda_{[001]}} \right) d\zeta = \frac{2}{\pi} \bar{\chi} \quad (32)$$

will be substituted into the left hand side of (31). The length of the half-period $\lambda_{[001]}/2$ has to be substituted for h . Finally, taking into account (26) and (28) leads to the following expression for the maximum bending amplitude of a stable crystalline undulator

$$a_s = (0.144 \text{ \AA}) \ln \left(\frac{\lambda_u}{2\sqrt{2} \text{ \AA}} \right). \quad (33)$$

It was found in experiment [36] that dislocation-free epilayers of much higher thickness and germanium content could be grown than it had been predicted by Matthews-Blakeslee formula (31). The reason for it was the kinetic barrier that had to be overcome before a dislocation could be formed. Therefore, the thickness of a metastable epitaxial layer can exceed the critical value for the stable layer by an order of magnitude or more. In fact, the Matthews-Blakeslee limit reveals itself only after annealing the specimen for about 30 min at 750–900 °C [35].

Several models describing the critical thickness (or, equivalently, the critical germanium content) of a metastable strained layer have been proposed [37–39]. The formula of People and Bean [37],

$$\chi_m^2 = \frac{10.9 \text{ \AA}}{h} \ln \left(\frac{h}{4 \text{ \AA}} \right), \quad (34)$$

will be used in the following. It agrees well with the experimental data [36] obtained for a $\text{Si}_{1-\chi}\text{Ge}_\chi$ layer grown by molecular beam epitaxy at 550 °C.

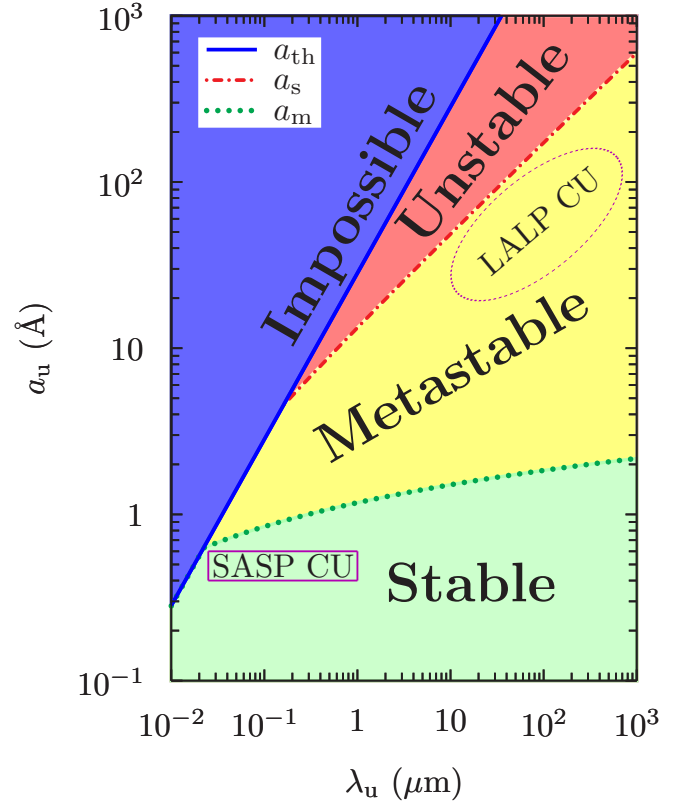


FIG. 2: The stability diagram of strained layer crystalline undulator based on graded $\text{Si}_{1-\chi}\text{Ge}_\chi$ composition vs. the bending period λ_u of the plane (01 $\bar{1}$) and the bending amplitude a_u (see the text for details). The crystalline undulator with a small amplitude and a short period (SASP CU) is predicted to be stable, while that with a large amplitude and a long period (LALP CU) is metastable.

Repeating the steps that led to formula (33) one obtains the maximum amplitude of a metastable crystalline undulator

$$a_m = \sqrt{(2.6 \cdot 10^{-3} \text{ \AA}) \lambda_u \ln \left(\frac{\lambda_u}{8\sqrt{2} \text{ \AA}} \right)}. \quad (35)$$

Equations (30), (33) and (35) are summarized in figure 2.

The region $a_u > a_{th}$ in the diagram is marked as ‘Impossible’. According to (26) and (29), undulators with such parameters would require the maximum germanium fraction to exceed 100%.

The region $a_m < a_u < a_{th}$ is called ‘Unstable’. The crystals with the corresponding germanium fraction can be grown, but its crystalline structure will be distorted by misfit dislocations. As a result, the desired shape of the crystal channel would not be obtained. Therefore, such crystals cannot be used as crystalline undulators.

The ‘Metastable’ region corresponds to $a_s < a_u < a_m$, a_{th} . Such crystals can be grown and the desired channel shape can be obtained. But the quality of such

crystalline undulators may degrade with time. The factors that facilitate the nucleation of dislocations, e.g. heat and ionizing radiation, may accelerate the aging of the strained layer crystal.

Finally, the undulators satisfying $a_u < a_s$, a_{th} are characterized as ‘Stable’. They are not expected to degrade with time. Moderately high temperatures, not very close to the melting point, are not expected to damage such undulators. Just in opposite, annealing may even improve their quality.

As is seen from the figure, SASP CU, having $a_u = 0.4$ – 0.6 Å [12] is located in the ‘Stable’ region of the diagram. In contrast, LALP CU, that requires $a_u \gg d = 1.92$ Å, can be only metastable.

To conclude, the crystalline undulator with a small amplitude and a short period (SASP CU) can be produced by growing a strained layer crystal of a graded $Si_{1-\chi}Ge_\chi$ composition. The germanium content χ can be varied according to equation (29). The average germanium content $\bar{\chi}$ has to be calculated using (26). The obtained strained layer crystal is predicted to be stable against misfit dislocations.

-
- [1] J. Lindhard, Kong. Danske Vid. Selsk. Mat.-Fys. Medd. **34**(14), 1–64 (1965).
 - [2] U. I. Uggerhøj, Rev. Mod. Phys. **77**, 1131 (2005).
 - [3] V. V. Kaplin, S. V. Plotnikov and S. A. Vorobiev, Zh. Tekh. Fiz. **50**, 1079-1081 (1980), (Sov. Phys. Tech. Phys. **25**, 650-651 (1980)).
 - [4] V. G. Baryshevsky, I. Ya. Dubovskaya and A. O. Grubich, Phys. Lett., **77A**, 61-64 (1980).
 - [5] Philip Willmott, “*An Introduction to Synchrotron Radiation: Techniques and Applications*”, John Wiley and Sons (2011).
 - [6] W. Greiner, A. V. Korol, A. Kostyuk, A. V. Solovyov, “Vorrichtung und Verfahren zur Erzeugung elektromagnetischer Strahlung”, German Patent DE102010023632, December 20 (2011).
 - [7] V.L. Ginzburg, Izv. Akad. Nauk. SSSR, Ser. Fiz. **11**, 165 (1947).
 - [8] H. Motz, J. Appl. Phys. **22**, 527, *ibid.* 1217 (1951).
 - [9] H. Motz, W. Thon, and R. N. Whitehurst J. Appl. Phys. **24**, 826 (1953).
 - [10] A. V. Korol, A. V. Solov'yov and W. Greiner, Int. J. Mod. Phys. E **13**, 867 (2004).
 - [11] M. A. Kumakhov, Phys. Lett. A **57**, 17 (1976).
 - [12] A. Kostyuk, arXiv:1210.0468 [physics.optics].
 - [13] S. Bellucci *et al.*, Phys. Rev. Lett. **90**, 034801 (2003).
 - [14] V. Guidi *et al.*, Nucl. Instrum. Meth. B **234**, 40 (2005).
 - [15] P. Balling *et al.*, Nucl. Instrum. Meth. B **267**, 2952 (2009).
 - [16] R. O. Avakian *et al.*, Nucl. Instrum. Meth. A **492**, 1113 (2002).
 - [17] A. Kostyuk *et al.*, Nucl. Instrum. Meth. B **266** (2008) 972.
 - [18] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
 - [19] U. I. Uggerhøj and E. Uggerhøj, Nucl. Instrum. Meth. B **160**, 435 (2000).
 - [20] S. C. Jain, J.R. Willis and R. Bullough, Adv. Phys. **39**, 127-190 (1990).
 - [21] S. C. Jain, S. Decoutere, M. Willander and H. E. Maes, Semicond. Sci. Technol. **16**, R51–R65 (2001).
 - [22] D. J. Paul, Semicond. Sci. Technol. **19**, R75–R108 (2004).
 - [23] M. B. H. Breese, D. G. de Kerckhove, P. J. C. King and P. J. M. Smulders Nucl. Instrum. Meth. B **132**, 177–187 (1997).
 - [24] M. B. H. Breese, Nucl. Instrum. Meth. B **132**, 540–547 (1997).
 - [25] H. Backe, Nuovo Cimento C, **34**(4), 157–165 (2011).
 - [26] L. Vegard, Z. Phys. **5**, 17 (1921).
 - [27] J. P. Dismukes, L. Ekstrom, and R. J. Paff, J. Phys. Chem., **68**, 3021–3027 (1964).
 - [28] L. D. Landau and E. M. Lifshitz, “*Theory of Elasticity*”, ButterworthHeinemann, Oxford etc. (1999).
 - [29] W. Krause *et al.*, Nucl. Instrum. Meth. A **483**, 455–460 (2002).
 - [30] R. Hull (ed.), “*Properties of Crystalline Silicon*”, IET, London (1999).
 - [31] C. Claeys (ed.) and E. Simoen (ed.), “*Germanium-Based Technologies: From Materials to Devices*”, Elsevier, Amsterdam (2007).
 - [32] J. H. Van der Merwe, J. Appl. Phys. **34**, 123–127 (1962).
 - [33] J. W. Matthews and A. E. Blakeslee, J. Cryst. Growth **27**, 118–125 (1974).
 - [34] S. C. Jain *et al.*, Philos. Mag. A **65**, 1151-1167 (1992).
 - [35] D. C. Houghton *et al.*, Thin Solid Films, **183**, 171–182 (1989).
 - [36] J. C. Bean *et al.*, J. Vac. Sci. Technol. A **2**, 436–440 (1984).
 - [37] R. People and J. C. Bean, Appl. Phys. Lett. **47**, 322–324 (1985); erratum: Appl. Phys. Lett. **49**, 229 (1986).
 - [38] B. W. Dodson and J. Y. Tsao, Appl. Phys. Lett. **51**, 1325–1327 (1987); erratum: Appl. Phys. Lett. **52**, 852 (1988).
 - [39] D. C. Houghton, Appl. Phys. **70**, 2136–2151 (1991).